associated with slab rollback is produced at the corner between the plates. It also matches the observation that narrow slabs roll back relatively quickly, because the flow around their edges significantly reduces the corner suction^{6,7}. The Ionian slab, which is narrower than it is long (Fig. 1), thus appears to be an excellent example of this process.

The truncated shape of this slab at its southern edge is clearly indicated by the abrupt cessation of seismic activity approximately at the north coast of Sicily. About 15 Myr ago this edge was torn away from another slab fragment, now detected by seismic tomography under the coasts of Algeria²³. The Ionian fragment rolled back to its present position, opening the Tyrrhenian backarc basin, whereas its other fragment sank in the mantle under the coasts of Algeria²⁴.

The shape of the northern edge of the Ionian slab is less clear. Earthquake distribution and tomographic studies seem to indicate that the Ionian slab is disconnected from the nearby Adriatic slab^{25,26} and that the Adriatic slab is decoupled from its mother plate, Adria^{26–28}. In Fig. 2a we suggest that here, the sinking Adriatic plate creates a local flow in the asthenosphere that produced the Quaternary magmatism at Mount Vesuvius and nearby. However, the exact subsurface geometry at this triple junction, between the Ionian, Tyrrhenian and Adriatic microplates, is yet to be explored.

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Decline in Mesozoic reef-building sponges explained by silicon limitation

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Several unrelated clades of siliceous sponges proliferated on the shelves of the Jurassic Tethys Sea, becoming prominent builders in reefs and near-shore mounds¹⁻⁴. Many of these builders are characterized by massive, rock-like skeletons made of spicules with a characteristic terminal hypersilicification^{4,5}. Such hypertrophied spicules are generically known as desmas, irrespective of their phylogenetic origin⁵. Desma-bearing sponges virtually disappeared from reefs and other neritic environments during the Cretaceous and the Early Tertiary^{1,2,4,6}, but have subsisted in relict populations in deeper, bathyal waters^{5,7,8}. The causes of the decline and bathymetric shift of these sponges remain obscure. Here we show experimentally that the concentration of silicic acid in seawater modulates the phenotypic expression of the various spicule types genetically available in a sponge species. We also show that the concentration of this nutrient in Recent surface waters is insufficient for this species to secrete its desmas. These findings indicate that silicon limitation, probably aggravated in shallow waters by the diatom burst around the Cretaceous-Tertiary boundary^{9,10}, may have forced neritic sponges with desmas to either lighten their skeletons or move to deeper, silicon-rich environments.

Silicon (Si) is one of the most abundant elements on earth, but only its soluble form, as monomeric, undissociated silicic acid, Si(OH)₄, is biologically assimilable. Several groups of marine organisms, such as diatoms, radiolarians, choanoflagellates and sponges, take up Si(OH)₄ from seawater to build their opal (amorphous hydrated silica) skeletons. Biological consumption of Si(OH)₄ renders present-day oceans undersaturated with this nutrient. Yet deep waters are relatively rich in Si(OH)₄, with concentrations varying from 10 to 180 μ M, whereas surface waters are notably poorer, with a mean concentration of less than 3μ M^{11,12}.

Diatoms are responsible for the depletion of Si(OH)₄ in the photic zone and largely control the cycling of Si in Recent oceans¹³. Radiolaria are quantitatively important only in the Pacific equatorial bell¹⁴, whereas siliceous sponges and silicoflagellates are considered unimportant on a global scale¹⁰. However, there is sedimentological evidence that sponges once controlled the Si cycling in neritic environments¹⁰. Indeed, desma-bearing sponges and hexactinellids, two unrelated groups with substantial siliceous

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skeletons, proliferated in neritic assemblages and became prominent builders in reefs and near-shore mounds during the Upper Jurassic¹⁻⁴. We show that the encrusting demosponge *Crambe crambe* (Schmidt), one of the most common macroinvertebrates in the western Mediterranean sublittoral¹⁵, provides some clues to understand why sponges with massive silica skeletons declined in neritic environments. In most natural populations, skeletons of *C. crambe* contain only a single spicule type, small needles called styles¹⁶; however, specimens are occasionally found in which the skeleton is supplemented by large styles, scarce C-shaped spicules (called isochelae) or scarce aster-like desmas¹⁶. This variability has puzzled sponge biologists, as the number of spicule types secreted by a species is a cornerstone character in sponge taxonomy; it is assumed that the spicule set does

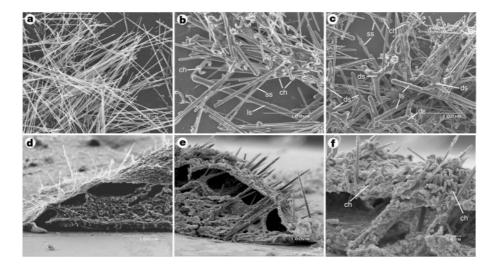


Figure 1 Skeletons produced under different concentrations of Si(OH)₄. **a**, Skeleton of a control sponge built up by a single, needle-like spicule type called style. **b**, Skeleton of a sponge reared in 30 μ M Si(OH)₄ showing three spicule types: small styles (ss); large styles (ls); and small C-shaped spicules called isochelae (ch). Note differences in size and robustness between styles of control sponges and styles of sponges reared in Si-enriched

seawater. **c**, Skeleton of a sponge reared in 100 μ M Si(OH)₄ showing four spicule types: styles in two size categories (ss, ls); isochelae (ch); and aster-like desmas (ds). **d**, **e**, Skeletal arrangements seen by fracturing sponges reared in controls and in 30 μ M Si(OH)₄, respectively. **f**, Ectosome of a sponge reared in 30 μ M Si(OH)₄ showing great abundance of protruding isochelae (ch).

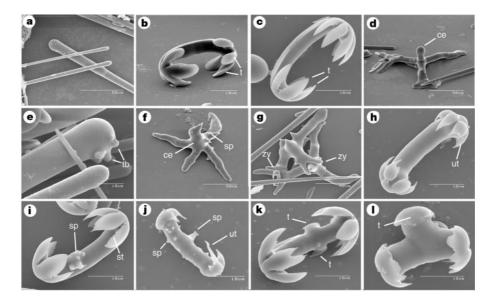


Figure 2 Spicules produced under two experimental levels of Si supply. **a**–**d**, Spicules produced in 30 μ M Si(OH)₄. **a**, Styles in two size categories. **b**, **c**, Isochelae showing intraspecific variability in the shape and number of teeth (t), traits assumed to have taxonomic value in some sponge groups. **d**, An aster-like desma with an undeveloped centrum (ce). **e**–**I**, Spicules produced in 100 μ M Si(OH)₄. **e**, Small and large styles, the latter with a tuberose ornamentation (tb), a trait of presumed taxonomic value. **f**, Aster-like

desma showing a well-developed centrum (ce) with three large spines (sp). **g**, Desmas still in contact after treatment with nitric acid, indicating some degree of fusion at the contact points; zy, zygoses. **h**–**I**, Common, malformed isochelae showing teeth reduced to ungues (ut) (**h**), serrated teeth (st) and spines (sp) on the shaft (**i**), reduced teeth (ut) and spines (sp) on the shaft (**j**), and teeth (t) on the shaft (**k**–**I**).

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not vary within species.

We examined the production of spicules by newly settled sponges maintained at 3 concentrations of Si(OH)₄ for 14 weeks (August to November 1998). One set of 36 sponges was cultured in seawater from their natural habitat. Concentration of Si(OH)₄ in these controls ranged from 0.03 to 4.5 μ M (mean \pm s.e. = 0.741 \pm 0.133 μ M). Two other sets were reared at elevated concentrations of Si(OH)₄, referred to hereafter as Simedium (30.235 \pm 0.287 μ M) and Si-high (100.041 \pm 0.760 μ M) treatments.

All sponges reared in control cultures formed only a single spicule type: small styles (Fig. 1a, d). In contrast, all sponges reared in Simedium conditions secreted at least three spicule types in great abundance: small styles, large styles and $28-33-\mu$ m long isochelae (Figs 1b, e–f, 2a–c). Fifty per cent of these sponges also formed a small number (1–5) of a fourth spicule type: aster-like desmas (Fig. 2d). In Si-high conditions, all sponges produced all four spicule types in great abundance (Figs 1c, 2e–l). Two of these sponges also secreted a fifth category: $8-10-\mu$ m long isochelae (spicule not shown). Notably, all sponges used in the experiments were from a large population of *C. crambe* studied for more than 15 years, in which we never found large styles, isochelae or desmas.

The concentration of Si(OH)₄ had a striking effect on both the size and shape of spicules (Figs 1a-c, 2). In Si-high conditions, a few isochelae formed spines and tooth-like structures at the middle of the shaft (Fig. 2k, l), which resembled spicules from the fossil record^{4,17}. Differences in Si(OH)₄ concentration also affected its uptake, as determined by analyses during the first five weeks of the experiment. Consumption curves of Si(OH)₄ were parallel in all three treatments, with a peak immediately after metamorphosis (Fig. 3). Sponges took up Si(OH)₄ more efficiently in Si-medium than in Si-high conditions, indicating that silica transport into skeletal secretory cells (sclerocytes) may be inhibited at unusually high concentrations. This may explain the production of numerous malformed isochelae in Si-high conditions (Fig. 2h-l). More significantly, Si(OH)₄ uptake was higher in the two Si-enriched treatments than in controls (Fig. 3), which indicates that C. crambe is limited by Si availability in coastal waters.

Our results show that *C. crambe* is genetically capable of producing spicule types that are not normally found in natural populations. Limited silicic acid in coastal waters appears to be the reason for this phenotypic skeletal inhibition, which affects entire populations of thousands of individuals. Our results also indicate that, in contrast to what is believed, each spicule type may be secreted by a specific sclerocyte type, which is activated by a particular Si(OH)₄ concentration threshold. Significantly, aster-like desmas are consistently formed only at concentrations of Si(OH)₄ (100 μ M) that are not found in most surface waters today. Sponges with aster-like desmas were fairly common in Jurassic–Cretaceous assemblages, but all these are now extinct, except the sublittoral genus *Crambe* and, perhaps, the monotypic, deep-water genus *Vetulina*^{4,16,18}.

The possibility that Si(OH)₄ concentration influenced the skeletal evolution of sponges is discussed elsewhere4,19. Two peaks of volcanic activity in the Triassic (Fassanian and Sevatian), which probably increased the amount of dissolved Si in seawater, may have favoured the radiation and proliferation of sponges with massive siliceous skeletons during the Jurassic^{4,19}. The hypothesis that seawater was saturated with dissolved Si at that time is also supported by the robust skeletons of fused spicules and state of preservation of fossils^{9,20}. The study of chert beds indicates that domination of the Si cycle by diatoms was completed only in the Eocene, after a long period of expansion and radiation beginning during the Cretaceous²¹. The contribution of sponge spicules to neritic cherts began to decline concurrently with the radiation and proliferation of diatoms9,10. The palaeontological record indicates that both desma-bearing and hexactinellid sponges had disappeared from neritic environments and penetrated the bathyal and abyssal zones by the Lower Tertiary^{1,4,6}. During the Cenozoic, siliceous spicules of neritic sponges became progressively less robust and fused spicules less frequent^{6,9}. The few Recent hexactinellids still found at shallow depths occur either in microhabitats with enhanced Si supply²² or at high latitudes^{23,24}, where the concentration of Si(OH)₄ in surface water is higher. There is also correlative evidence that the expansion of diatoms reduced both diversity and abundance of other Sidependent organisms, such as radiolarians²⁵.

The hypothesis that low Si(OH)₄ concentration in shallow waters might limit sponges has been disregarded^{7,26,27}, as the Si-uptake system of sponges is expected to have adapted to the decline in the availability of this nutrient. In contrast, our results indicate that Si concentration is a limiting factor on shallow-water sponges and that this pressure may be strong enough to shape their skeletons and restrict their bathymetric distributions. Chronic Si limitation is also supported by the only study on Si uptake in a sublittoral sponge, Halichondria panicea. This showed Michaelis-Menten-uptake kinetics with saturation at Si(OH)₄ concentrations far higher than the maximum available in the natural habitat of the species²⁸. We propose that chronic limitation may be responsible not only for much of the unexplained skeletal variability that puzzles sponge taxonomists, but also for much of the skeletal change observed in shallow-water sponges during the Cenozoic. In which sponge groups the 'skeletal change' represents evolution, and in which it merely reflects phenotypic inhibition of certain spicule types remain unclear. Therefore, we cannot rule out the possibility that by rearing more sponges in seawater saturated with silicic acid some apparently extinct species or spicules may 'come back to life' for taxonomists.

Methods

The sponges used in the experiment were grown in the laboratory from free-swimming

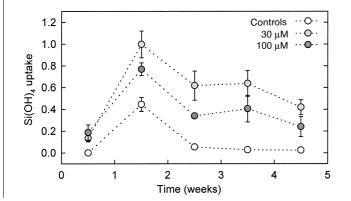


Figure 3 Consumption (mean \pm s.e.) of Si(OH)₄ by sponges (µmol per individual per week) as a function of substrate concentration and time during the first five weeks of the experiment.

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larvae collected randomly in the field (2° 48.12' N, 41° 40.33' E) by SCUBA. Between 5 and 10 juveniles were recruited successfully in each of 15, 11 polystyrene containers (n = 15), the bottom of which was covered with an acetate sheet that served as substratum for sponge attachment. Containers were then randomly distributed in 3 groups, and sponges in each group were reared for 14 weeks in 3 different concentrations of Si(OH)₄: $0.741\pm0.133,\,30.235\pm0.287$ and 100.041 \pm 0.760 μM (mean \pm s.e.). All cultures were prepared using 0.22 µm polycarbonate-filtered seawater, which was collected from the sponge habitat, handled according to standard methods to prevent Si contamination²⁹ and enriched in dissolved silica, when treatments required, by using Na2SiF6. During the experiment, all sponges were fed by weekly addition of 2 ml of a bacterial culture $(40-60 \times 10^6 \text{ bacteria ml}^{-1})$ to each container³⁰. The seawater was replaced weekly, with regeneration of initial food and Si(OH)4 levels. The concentration of Si(OH)4 in cultures was determined on 3 replicates of 1 ml seawater samples per container by using a Bran-Luebbe TRAACS 2000 nutrient autoanalyser. After week 5, the accidental contamination of some culture containers by diatoms rendered subsequent estimates of Si uptake by sponges unreliable, so we discarded them for the study.

For the study of the skeleton, sponges were treated according to standard methods³⁰ and examined in a Hitachi S-2300 scanning electron microscope (SEM).

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Learning the parts of objects by non-negative matrix factorization

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Is perception of the whole based on perception of its parts? There is psychological¹ and physiological^{2,3} evidence for parts-based representations in the brain, and certain computational theories of object recognition rely on such representations^{4,5}. But little is known about how brains or computers might learn the parts of objects. Here we demonstrate an algorithm for non-negative matrix factorization that is able to learn parts of faces and semantic features of text. This is in contrast to other methods, such as principal components analysis and vector quantization, that learn holistic, not parts-based, representations. Non-negative matrix factorization is distinguished from the other methods by its use of non-negativity constraints. These constraints lead to a parts-based representation because they allow only additive, not subtractive, combinations. When non-negative matrix factorization is implemented as a neural network, parts-based representations emerge by virtue of two properties: the firing rates of neurons are never negative and synaptic strengths do not change sign.

We have applied non-negative matrix factorization (NMF), together with principal components analysis (PCA) and vector quantization (VQ), to a database of facial images. As shown in Fig. 1, all three methods learn to represent a face as a linear combination of basis images, but with qualitatively different results. VQ discovers a basis consisting of prototypes, each of which is a whole face. The basis images for PCA are 'eigenfaces', some of which resemble distorted versions of whole faces⁶. The NMF basis is radically different: its images are localized features that correspond better with intuitive notions of the parts of faces.

How does NMF learn such a representation, so different from the holistic representations of PCA and VQ? To answer this question, it is helpful to describe the three methods in a matrix factorization framework. The image database is regarded as an $n \times m$ matrix V, each column of which contains *n* non-negative pixel values of one of the *m* facial images. Then all three methods construct approximate factorizations of the form $V \approx WH$, or

$$V_{i\mu} \approx (WH)_{i\mu} = \sum_{a=1}^{r} W_{ia} H_{a\mu}$$
(1)

The *r* columns of *W* are called basis images. Each column of *H* is called an encoding and is in one-to-one correspondence with a face in *V*. An encoding consists of the coefficients by which a face is represented with a linear combination of basis images. The dimensions of the matrix factors *W* and *H* are $n \times r$ and $r \times m$, respectively. The rank *r* of the factorization is generally chosen so that (n + m)r < nm, and the product *WH* can be regarded as a compressed form of the data in *V*.

The differences between PCA, VQ and NMF arise from different constraints imposed on the matrix factors W and H. In VQ, each column of H is constrained to be a unary vector, with one element equal to unity and the other elements equal to zero. In other words, every face (column of V) is approximated by a single basis image (column of W) in the factorization $V \approx WH$. Such a unary encoding for a particular face is shown next to the VQ basis in Fig. 1. This unary representation forces VQ to learn basis images that are prototypical faces.